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2. Braess, D., *finite Elements*, 3<sup>rd</sup> edition, Cambridge University Press, Cambridge, 2007.

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# Discontinuous Fractal Interpolation

M.A. Navascués\*

## Abstract

The fractal interpolation functions provide curves whose graph has generally a non-integer dimension. They own other characteristics as the interpolation of a set of data and the continuity. In this paper, the latter conditions are omitted, defining discontinuous fractal functions passing close to (but not necessarily through) the given data.

In a second part of the article we define affine fractal functions not linked to two-dimensional data. To do this we use the methodology of iterated functions systems. They are composed of a finite set of contractive affinities whose attractor is related to the graph of a bounded function. In this way the paper introduces a very large class of affine fractal functions which are generally discontinuous (though they contain the classical continuous case as a particular case) and whose relevance is not based only on the approximation.

**Keywords:** Fractals, Discontinuous functions, Interpolation, Approximation

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# Popoviciu Type Inequalities Via Green Function and Hermite's Polynomial

Saad Ihsan Butt\*, Ram N. Mohapatra<sup>†</sup> and Josip Pečarić<sup>‡</sup>

## Abstract

The Hermite polynomial and Green function are used to construct the identities related to Popoviciu type inequalities for higher order convex functions. We investigate the bounds for the identities related to the generalization of the Popoviciu inequality using inequalities for the Čebyšev functional. Some results relating to the Grüss and Ostrowski type inequalities are constructed. Further, we also construct new families of exponentially convex functions and Cauchy-type means by looking at linear functionals associated with the obtained inequalities.

**Keywords:** Convex Function, Divided Difference, Generalized Montgomery Identity, Čebyšev Functional, GrÜss Inequality, Ostrowski Inequality, Exponential Convexity.

## 1 Introduction and Preliminary Results

A characterization of convex function established by T. Popoviciu [18] is studied by many people (see [19, 17] and references with in). For recent work, we refer [7, 10, 11, 14, 15]. The following form of

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# Applications of Exponential Convexity

Asfand Fahad\*, Josip Pečarić<sup>†</sup> and Julije Jakšetić<sup>‡</sup>

## Abstract

In this paper, we apply  $n$ -exponential convexity and log-convexity on a positive linear functional defined as the difference of the left hand side and right hand side of the inequalities from [3]. We obtain interesting inequalities and improvements of Hardy type inequality given in [3].

**Keywords:** Convex Function, Divided Difference, Generalized Montgomery Identity, Čebyšev Functional, GrÜSs Inequality, Ostrowski Inequality, Exponential Convexity.

## 1 Introduction and Preliminaries

Steffensen [12] proved the following inequality: if  $f, h : [\alpha, \beta] \rightarrow \mathbb{R}$ ,  $0 \leq h \leq 1$  and  $f$  is decreasing, then

$$\int_{\alpha}^{\beta} f(t)h(t) dt \leq \int_{\alpha}^{\alpha+\gamma} f(t) dt, \quad \text{where } \gamma = \int_{\alpha}^{\beta} h(t) dt. \quad (1.1)$$

Several papers are devoted to studying generalizations of Steffensen's inequality (1.1). Convex functions are used in some generalization of Steffensen inequality. One recent generalization is given by Rabier [11].

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# **Solution of Reaction Diffusion Problem using Homotopy Perturbation Method and Differential Transformation Method: A Comparative Study**

Nini Maharana,<sup>\*</sup> A. K. Nayak,<sup>†</sup> H. B. Pattnaik,<sup>‡</sup> and S. Srivastav<sup>§</sup>

## **Abstract**

In recent years, a new difference scheme with high accuracy has been applied for solving convection-diffusion equation [3]. In this paper an application of Homotopy perturbation method (HPM) is used to solve linear and non-linear diffusion-reaction problem (NDRP). Diffusion-Reaction equations have special importance in engineering and sciences and constitute a good model for many systems in various fields. We tried to compare the differential transform method (DTM) and HPM for solving time dependent reaction-diffusion equations and found that the proposed method HPM are comparable with the results of DTM for small parameter values but differed at large parameters. The proper implementation of He's Homotopy perturbation method can extremely minimize the size of work if compared with the existing differential transformation method.

**Keywords:** Homotopy Perturbation Method, Differential Transformation Method, Reaction Diffusion Problem.

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# Some Geometric Studies on the Classes of Bi-Univalent Functions

Pravati Sahoo\*, R. N. Mohapatra<sup>†</sup>

## Abstract

This article is a survey, in which we analyze certain aspects of the class of bi-univalent complex-valued functions defined on the unit disk. After the appearance of the paper by Lewin in the 1967, the class of bi-univalent functions did begin to attract interest among function theorists. He proposed a coefficient conjecture for the class of bi-univalent analytic functions like Bieberbach. In this article we begin with the basic definitions and some examples of bi-univalent functions. After a brief look at the literature, we focus our attention on the coefficient bounds for several geometric subclasses and discuss the recent developments along this line.

**2010 MSC:** 30C45, 30C50

**Keywords:** Univalent Functions, Bi-Univalent Functions, Starlike and Bi-Starlike Functions, Coefficient Bounds, Faber Polynomial.

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# Fixed Points for Contractive Mappings on a Metric Space with a Graph: A Survey

Asrifa Sultana\*and V.Vetrivel<sup>†</sup>

## Abstract

In this paper we discuss and relate some important fixed point theorems and best proximity point theorems for contractions on a metric space endowed with a graph proved by various authors in recent times. We establish an existence theorem on best proximity point for generalized contractive mappings on a metric space endowed with a graph. Moreover, our theorem subsumes and generalizes many recent fixed point and best proximity point results.

**2010 MSC:** 54H25, 47H10

**Keywords:** Fixed Point, Set-Valued Map, Best Proximity Point, Contraction, Graph, Metric Space,  $P$ -Property

## 1 Introduction

Fixed point theory plays an important role in supplying a uniform treatment for solving equations of the form  $f(x) = x$  where  $f$  is a mapping from a set  $K$  into a set  $X$  containing  $K$ . An element  $x \in K$  is said

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# Generalized Convexity in Mathematical Programming

S. Nanda\* and N. Behera<sup>†</sup>

## Abstract

The purpose of the paper is to give a brief review of some generalized convex functions existing in the literature. It also contains some unpublished definitions and results.

**Keywords:** Convex Function, Convexity, Mathematical Programming.

**2010 AMS classification:** 90 C 30.

## 1 Introduction

Convexity plays a key role in mathematical programming. Though many significant results in mathematical programming have been derived under convexity assumptions, yet most of the real world problems are nonconvex in nature. Therefore a systematic attempt is being made by several authors to introduce and discuss various new kinds of generalized convex functions.

The purpose of this note is to give a brief review of various generalizations of convexity existing in the literature. The definitions of generalized convex functions are given in a tabular form and some results are quoted which give relationship among these concepts. This review also mentions some unpublished works of the authors and suggests several open problems for further study.

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